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THE BAUSCHINGER EFFECT IN AUTOFRETTAGED TUBES - A COMPARISON OF MODELS INCLUDING THE ASME CODE

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Autofrettage is used to introduce advantageous residual stresses into pressure vessels and to enhance their fatigue lifetimes. For many years workers have acknowledged the probable influence of the Bauschinger effect which serves to reduce the yield strength in compression as a result of prior tensile plastic overload. This in turn can produce lower compressive residual hoop stresses near the bore than are predicted by 'ideal' solutions (elastic/perfectly plastic without Bauschinger effect).

There have been several models proposed in order to predict the reduced stresses within the autofrettaged tube. The purpose of this paper is simply to compare a limited set of models, including the ASME code, with available experimental evidence. Three models are compared: Model A, based upon a quasi strain-hardening model developed by Chen; Model B, based upon a Bauschinger effect which varies with plastic strain and hence with radius; Model C, which is based upon section KD-522.2 of the recently revised ASME pressure vessel code. The models are compared against experimental data under three headings:

- Measurements of Hoop Residual Stress at the Bore
- Measurements of Hoop Residual Stress Variation Radially Through the Tube Wall, In Particular the Near-Bore Region
- Measurements of Opening Angle When Autofrettaged Tubes are Slit Radially, Hence Releasing the Pure Bending Moment 'Locked In' by the Hoop Stress

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ABSTRACT

Autofrettage is used to introduce advantageous residual stresses into pressure vessels and to enhance their fatigue lifetimes. For many years workers have acknowledged the probable influence of the Bauschinger effect which serves to reduce the yield strength in compression as a result of prior tensile plastic overload. This in turn can produce lower compressive residual hoop stresses near the bore than are predicted by 'ideal' solutions (elastic/perfectly plastic without Bauschinger effect).

There have been several models proposed in order to predict the reduced stresses within the autofrettaged tube. The purpose of this paper is simply to compare a limited set of models, including the ASME code, with available experimental evidence. Three models are compared; Model A, based upon a quasi strain-hardening model developed by Chen; Model B, based upon a Bauschinger effect which varies with plastic strain and hence with radius; Model C, which is based upon section KD-522.2 of the recently revised ASME pressure vessel code. The models are compared against experimental data under three headings:

Measurements of Hoop Residual Stress at the Bore - For design purposes, a lower (conservative) bound is sought. In the case of the bore residual stress data Model B, based upon 0.1% offset data, clearly provides such a bound.

Measurements of Hoop Residual Stress variation radially through the tube wall, in particular the near-bore region -Model B predicts, near the bore, a hoop stress which decreases with increasing radius; conversely Models A and C predict an ever-increasing hoop stress. Available X-ray diffraction results appear to provide confirmation of a reduction.

Measurements of Opening Angle when autofrettaged tubes are slit radially hence releasing the pure bending moment 'locked in' by the hoop stress - The comparison of tube slitting results is less definitive but appears to indicate that Model B, based upon 0.1% offset data, provides a suitable lower bound.

The three models were used to predict fatigue lifetime for cyclically pressurized thick cylinders with pre-existing cracks. The plots indicate reasonable agreement between the three models up to 40% overstrain, but significant disagreement at high overstrain levels with almost an order of magnitude discrepancy at 100% overstrain between the lifetime predictions of Models B and C.

Taken together the above comparisons indicate some significant areas of disagreement between the three models. In the cases of residual stress at the bore and near the bore, Models A and C are both potentially non-conservative. Whilst systematic experimental evidence is not available in relation to fatigue lifetimes, use of Model C, section 522.2 of the ASME code, in isolation from other sections of the Code, could result in a very significant over-estimate of such lifetimes. Until additional experimental evidence becomes available the authors recommend the use of Model B based upon 0.1% offset data.

INTRODUCTION

Autofrettage is used to introduce advantageous residual stresses into pressure vessels and to enhance their fatigue lifetimes.

For many years workers have acknowledged the probable influence of the Bauschinger effect (Bauschinger, 1881) which serves to reduce the yield strength in compression as a result of prior tensile plastic overload. This phenomenon is illustrated in Fig. 1(a) wherein the yield strength in tension is Y and the yield strength in compression is -fY; the data in Fig. 1(a) are based upon work by Clark (1982). f is sometimes termed the *Bauschinger Effect Factor* (BEF); work by Milligan et al. (1966) provides a relationship between tensile plastic overstrain and the BEF; the latter varies from unity at zero plastic strain, drops rapidly with increasing plastic strain and saturates at around 2% plastic strain, being effectively constant thereafter. This saturation value of BEF is designated f*. The variation of BEF, based upon Milligan et al. (1966), is illustrated in Fig. 1(b).

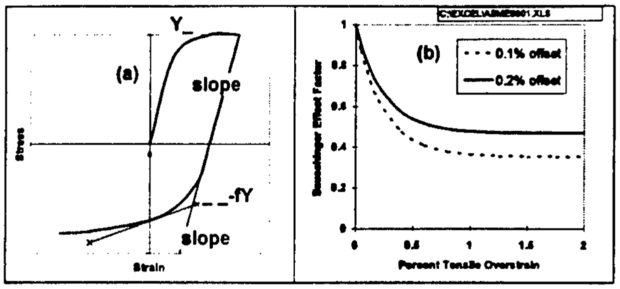

Figure 1 : Bauschinger Effect, Relevant Earlier Work:
(a) Stress-Strain Curve for Typical Gun Steel, after Clark (1982)
(b) Variation of BEF with Percentage Plastic Strain, after Milligan et al. (1966)

The reduction of compressive yield strength within the yielded zone of an autofrettaged tube is of importance because, on removal of the autofrettage pressure, the region near the bore experiences high values of compressive hoop stress, approaching the normal tensile yield strength of the material if the unloading is totally elastic. If the combination of hoop and radial stresses exceeds some yield criterion the tube will re-yield from the bore thus losing much of the potential benefit of autofrettage.

There have been several models proposed in order to predict the reduced stresses within the autofrettaged tube. The common feature of these models is the partial inclusion of data from Milligan et al. (1966). Chen (1986) incorporated a constant value of f based upon plastic strain at the bore and the continued 'compressive strain hardening' slope, also illustrated in Fig. 1(a). Chen employed a constant value for the slope, m', of 0.3. In a recent paper (Parker and Underwood, 1998) the authors proposed a simple model incorporating the variability of f with radius arising from the variation of plastic strain with radius without any effect of strain-hardening. Section KD-522.2 of the recently revised ASME pressure vessel code (ASME, 1997) relates to Bauschinger effect corrections for autofrettaged tubes.

It is not the purpose of this paper to reproduce the analyses, which focus upon material behaviour within the Bauschinger Affected Zone (BAZ), i.e. the area near the bore which experiences reversed yielding upon removal of the original autofrettage bore pressure (hydraulic autofrettage) or displacement (swage autofrettage). Such detail is contained in Chen (1986) and Parker and Underwood (1998).

The purpose of this paper is simply to compare the limited set of models, including the ASME code, with available experimental evidence. Three models are compared: *Chen's model* (Chen, 1986); *ASME model* (ASME,1997); *Authors' model* (Parker and Underwood, 1998).

The following notation is used: inner radius of tube, a; outer radius of tube, b; maximum radius to which yielding extends during autofrettage, c; maximum radius to which reversed yielding extends during unloading, d.

SOURCES OF EXPERIMENTAL EVIDENCE

Experimental data on autofrettaged tubes was sought under four headings:

a. Measurements of Hoop Residual Stress at the Bore

Data are available: (Clark, 1982); (Lee et al., 1994); (Stacey and Webster, 1984); (Frankel et al., 1993). Fortuitously even though such data relates to a variety of tube radius ratios, b/a, and percentage overstrains, 100x(c-a)/(b-a), it is analytically appropriate to plot such data against a common parameter, c/a regardless of radius ratio (Parker and Underwood, 1998).

b. Measurements of Hoop Residual Stress variation radially through the tube wall, in particular the near-bore region.

Far more limited results are available in this area, indeed the only group which appears consistently to acquire large numbers of stress data readings in the near-bore region consists of Lee and co-workers; Lee et al. (1994) and Lee et al. (1997).

c. Measurements of Opening Angle when autofrettaged tubes are slit radially hence releasing the pure bending moment 'locked in' by the hoop stress (Parker and Underwood, 1998).

In the case of tube opening it is again fortuitous that a single line plot comparison is possible (Throop et al., 1982). Hence, although data relates to various wall ratios and overstrains a normalized presentation of 'ideal' (non-Bauschinger) tube opening angle versus percentage overstrain follows a single sigmoidal curve to within 1% for radius ratios 1.8<=b/a<=2.2.

d. Fatigue Lifetime Data (pre-existing crack-like defects)

Such data are available. However the sensitivity of fatigue lifetime to initial defect size, crack spacing and crack shape would render such comparisons highly dubious unless based upon a systematic set of tests in which such parameters were controlled whilst percentage overstrain was varied. Perhaps surprisingly such systematic data are not available and such a comparison is not attempted herein. It is however possible to predict the effect of the various models upon fatigue lifetime with controlled parameters and such a comparison is presented later in this paper.

COMPARISONS

Figure 2 shows hoop residual stress at the bore normalized with yield stress as a function of c/a. Lines represent the various models and points represent experimental data. The two lowest continuous lines show ideal values based upon the standard ideal elastic-plastic analysis using Tresca's criterion (Parker and Underwood, 1998) and Chen (1986) for b/a=1.8 and 2.0. The broader continuous line of discontinuous slope shows the ASME code (ASME, 1997) predictions

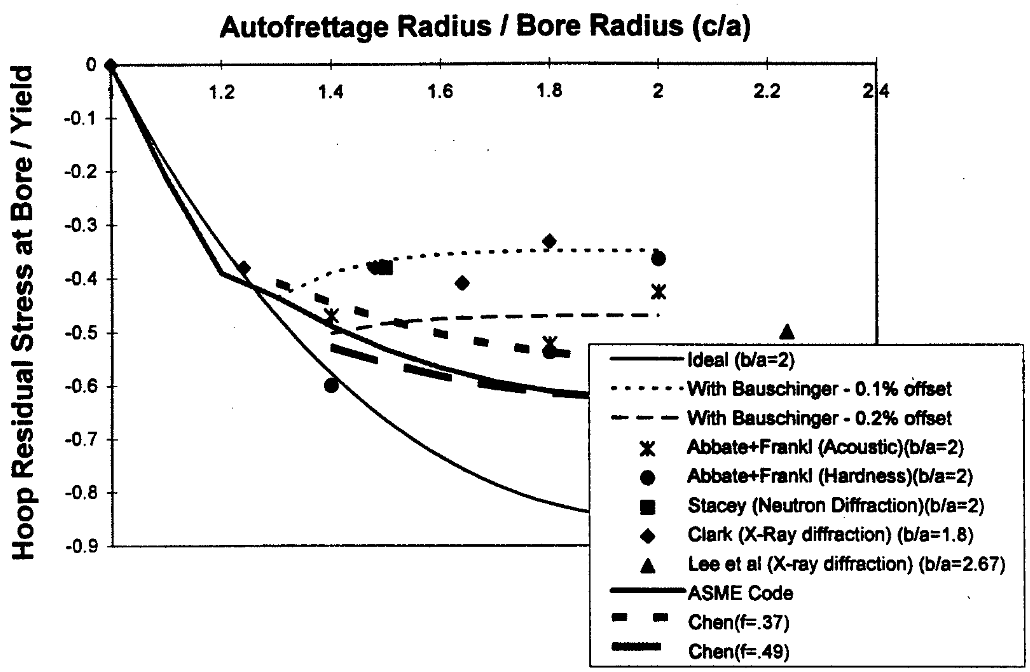

Figure 2 : Residual Stress at Bore - Bauschinger Models and Experimental Evidence

for b/a=2. The code employs these same (Tresca) stresses multiplied by 1.15 to simulate Von Mises' solution prior to the onset of reversed yielding. The slope discontinuity is associated with this reversed yielding onset.

The two heavy, dotted lines show the predictions of Chen's model using his recommended value of m'=0.3 and bounding values for f* of 0.37 and 0.49 associated with 0.1% and 0.2% offsets respectively. Similarly the two lighter dotted lines show predictions from the authors' model for 0.1% and 0.2% offsets. Experimental results from several separate sources are presented; Clark (1982); Lee et al. (1994); Stacey and Webster (1984); Frankel et al. (1993), encompassing techniques based upon acoustics, hardness, neutron diffraction and X-ray diffraction.

Figure 3 shows an averaged, equilibriated fit to the X-ray diffraction results of Lee et al. (1994) obtained for a tube having a = 57 mm, b = 152.4 mm, yield strength 1200 MPa with 74% overstrain. The two pairs of lines of discontinuous slope show the predictions of Chen's model and the authors' model for 0.1% and 0.2% offsets, whilst the remaining line shows the prediction of the ASME code.

Figure 4 shows, as a continuous line, the ideal angle of opening of an autofrettaged tube free of Bauschinger effect which has been cut radially. The line shown is for b/a=2 but is asymptotic at zero and 100% overstrain for all values of b/a <=2.22 and shifts by no more than 1% for 1.8<=b/a<=2.2.

Sets of experimental data points relating to 48% and 80%overstrain were obtained from recent cannon tube tests; 30% and 60% overstrain and 50%, 75% and 100% overstrain were obtained fromThroop et al. (1982). Note that the latter results have been carefully re-analyzed so that the technique for measuring opening angle (based upon datum markings prior to tube slitting) is consistent across all results in Fig. 4.

DISCUSSION OF EXPERIMENTAL EVIDENCE

In all cases we seek, for design purposes, a lower (conservative) bound. In the case of the bore residual stress data presented in Fig. 2 the authors' model based upon 0.1% offset clearly provides such a bound.

The authors' model predicts, within the BAF, a compressive hoop stress which decreases with increasing radius; conversely the Chen and ASME models predict an ever-increasing hoop stress. The X-ray diffraction results of Lee et al. (1994), Fig. 3, appear to provide confirmation of a reduction. It should be noted that Lee's raw data have been shifted vertically in order to ensure equilibrium, however such shifting cannot influence slopes.

The tube slitting results presented in Fig. 4 are less definitive but appear to indicate that the 0.1% offset model provides a suitable lower bound.

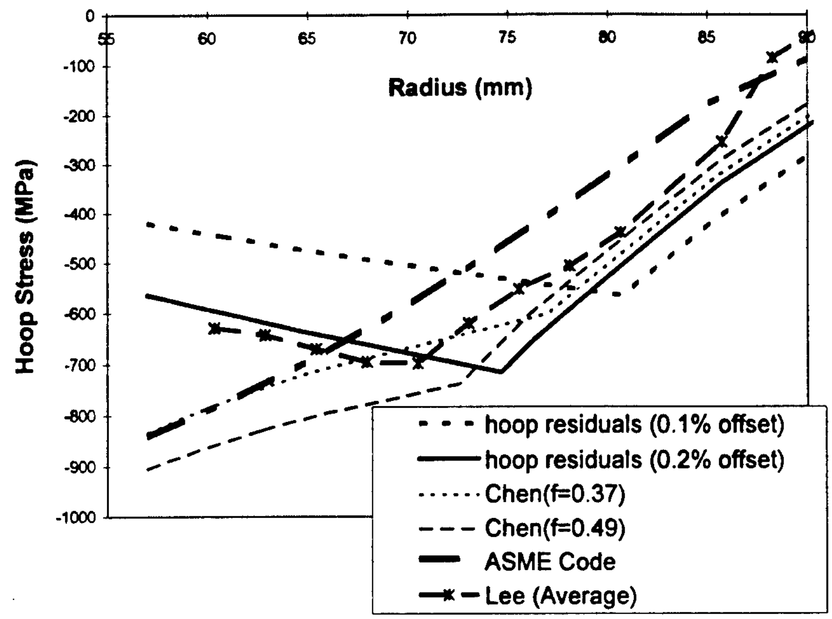

Figure 3 : Thru-Thickness Hoop Stress Bauschinger Models and Experimental Evidence from Lee et al. (1994)

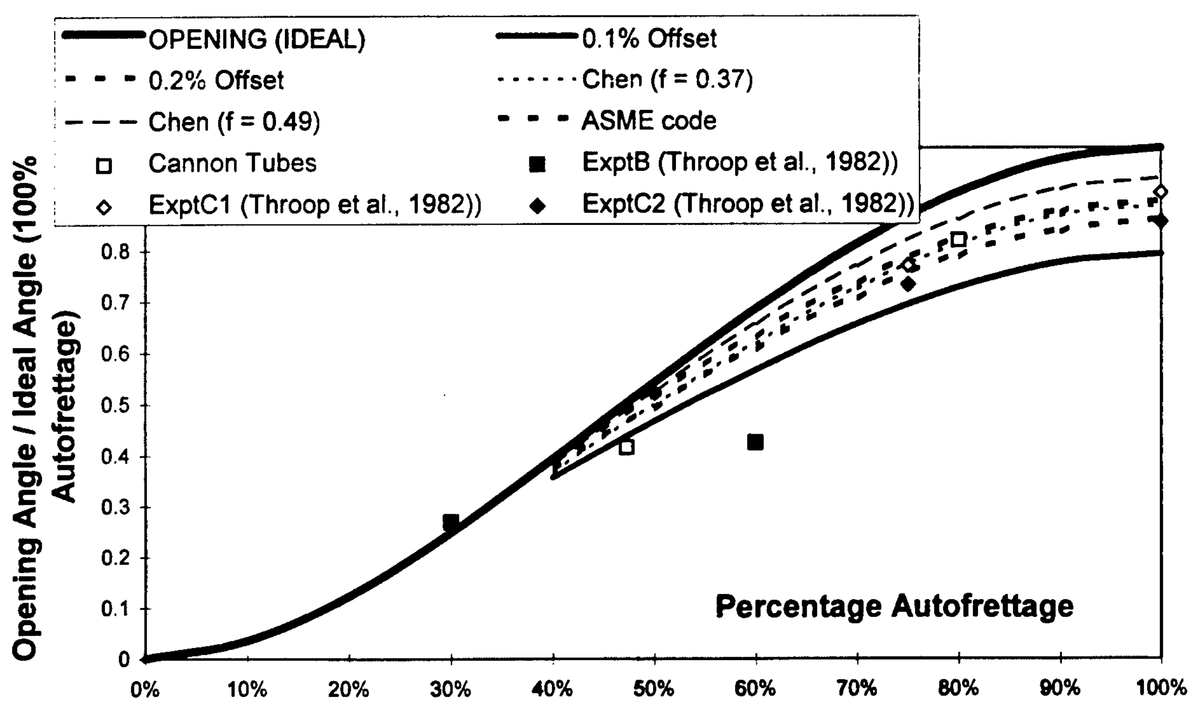

Figure 4 : Angular Opening of Radially Sliced Tubes - Bauschinger Models and Experimental Evidence

PREDICTION OF EFFECT OF VARIOUS MODELS UPON FATIGUE LIFETIME

The lifetime prediction technique employed in Parker and Underwood (1998) has been used to predict lifetimes via the ASME model and the authors' model.

Fatigue lifetimes, based upon stress intensity factor solutions of extremely high accuracy (errors < 0.5%) determined by the Modified Mapping Collocation technique (Andrasic and Parker, 1984) and packaged as weight function data (Andrasic and Parker, 1982), are presented in Fig. 5. The calculations were based upon the following geometrical and materials properties: a = 50 mm; b = 100 mm; four initial, straight-fronted diametrically opposed bore cracks of length 0.5 mm; internal cyclic pressure 400 MPa; Young's modulus, E, 200 GPa; Yield Strength 1200 MPa; Paris Law coefficient, C, 6.52E-12; Paris Law exponent, m = 3. The stress intensity factor calculations take full account of thru-the-thickness variation of residual and pressurization stresses. Overstrains from 0 to 100% were examined, and lifetimes calculated for the cases of ideal autofrettage (both Tresca and Von Mises) and incorporating Bauschinger effect (ASME model) and incorporating both 0.1% and 0.2% offsets (Chen's model and authors' model). The stationary points which appear for 0.1% and 0.2% offsets are real effects arising in the analysis and are related to the depth and shape of residual stresses in the Bauschinger affected zone and their subsequent effect upon stress intensity and hence fatigue lifetime.

The plots indicate reasonable agreement between models up to 40% overstrain, but significant disagreement at high overstrain levels with almost an order of magnitude discrepancy at 100% overstrain between the lifetime predictions of the authors' 0.1% offset model and those of the ASME model.

LINEAR UNLOADING COMPARISON

The most significant differences between the Chen and ASME models that incorporate strain hardening and the authors' model that does not would be for significant amounts of compressive plastic strain as the compressive residual stress is created during unloading. If there is significant compressive plastic strain, then the Chen and ASME models would be more appropriate. If there is only limited compressive plastic strain during unloading then the authors' model would be better. The question of the amount of plastic strain during unloading is not an easy one, because the materials and overstrain processes of various users vary, and any broad based modeling of these differences is a major task. However some answers to the question of the amount of compressive plastic strain during unloading can be obtained by using overstrain conditions typical of one important application, the mandrel overstrain of ASTM A723 steel thick-wall cylinders for cannon tubes. Typical cannon material and overstrain values are used in a classic linear unloading analysis of the cylinder inner diameter location, including an account of the Bauschinger effect, in the following discussion.

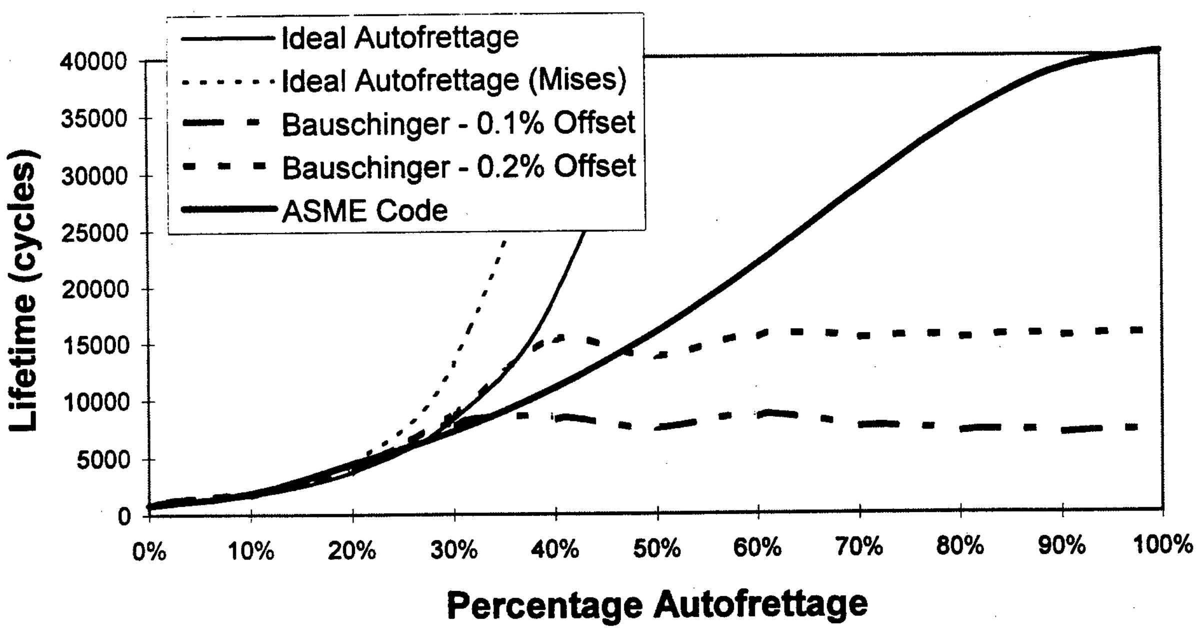

Figure 5 : Predicted Lifetimes as a Function of Percentage Autofrettage

Material properties and tube and mandrel configurations typical of cannon overstrain are as follows:

Yield strength;	1,200 MPa
Elastic modulus;	200,000 MPa
Inner and outer diameters;	150 mm; 290 mm
Mandrel diametral interference;	1.8 mm

Using these values and referring to a linear unloading analysis shown in Fig. 6, critical values of strain and the BEF can be determined as:

Yield strain, e_Y;	0.6 %
Overload elastic strain, e_{OV};	1.2 %
Overload plastic strain, e_P;	0.6 %
BEF; 0.2 % offset	0.53

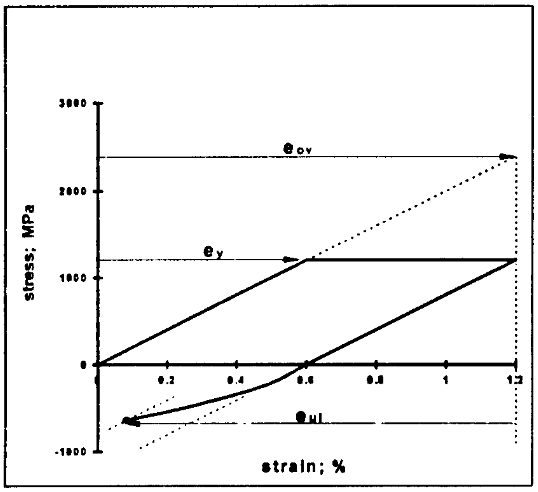

Figure 6 : Linear Unloading Model of Overstrained Cylinder ID Including Bauschinger Effect

In the above e_{OV} is the maximum effective elastic strain available for unloading (assuming a rigid mandrel) , and the BEF is determined from the Milligan et al. (1966) results in Fig. 1 for $e_P = 0.6$ %. The compressive portion of the unloading is determined from the BEF and yield offset values in Fig. 1. For example, the end point value of unloading shown in Fig. 6 is at -636 MPa (BEF x 1200 MPa) and 0.2 % offset from the elastic unloading curve.

The important point of this linear unloading analysis is that at the end point of the unloading shown in Fig. 6 nearly all of the maximum available elastic unloading strain has been used and yet the end point is barely displaced from the elastic unloading curve. A calculation that demonstrates this same important point is:

$$e_{UL} = e_Y + \text{BEF} \times e_Y + \text{yield offset} = 1.12\,\% \qquad (1)$$

Thus it is clear, for linear unloading analysis of cannon overstrain conditions, that very little compressive plastic strain during unloading is possible. Therefore, the authors' model of residual stresses and associated fatigue lives described here, that assumes no strain hardening, is quite appropriate. Moreover, the incorporation of significant strain hardening in models of overstrain residual stresses would, for some conditions at least, give higher than actual values of compressive residual stress and higher fatigue lives.

CONCLUSIONS AND RECOMMENDATIONS

The aggregated models and associated comparisons of experimental results indicate some significant areas of disagreement between the models. In the cases of residual stress at the bore and near the bore the ASME code and Chen's model are both potentially non-conservative. Whilst systematic experimental evidence is not available in relation to fatigue lifetimes, use of the ASME code distributions could potentially result in a very significant over-estimate of such lifetimes.

Until such time as additional experimental evidence becomes available the authors recommend the use of the author's model based upon 0.1% offset.

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Discussion of a paper "Influence of the Bauschinger Effect on Residual Stress and Fatigue Lifetimes in Autofrettaged Thick-Walled Cylinders",by A.P. Parker and J.H. Underwood

By David P. Kendall, Consultant, Troy, NY

The paper by Parker and Underwood points out a possible error in Division 3 of Section VIII of the ASME Boiler and Pressure Vessel Code. They show that the empirical method used in the code to correct for the Bauschinger Effect, when calculating autofrettage residual stresses, may be non-conservative. However, their results may be overly conservative because they do not account for the strain hardening that occurs during the reverse yielding that results from the Bauschinger Effect. The purpose of this discussion is to present an approximate method to account for this strain hardening and to compare the results of this analysis with the Parker and Underwood results.

Milligan et al [1] gave curves of compressive yield strength at various offsets, after various amounts of tensile prestrain for an A-723 steel. From these results, a series of compressive stress-strain curves as a function of tensile prestrain can be calculated. These curves can be fit with a set of equations that very accurately represent the Milligan, et al, results.

For $\epsilon_c > 0.1\%$,

$$\sigma_c = A + B\,\epsilon_c + C\,\epsilon_c^2 \quad \text{.......................................} (1)$$

where

$$A = 9.01\,\epsilon_p - 1.843\,\epsilon_p^2 + 4.74\,\epsilon_p^{0.1}$$
$$B = 300 - 210.2\,\epsilon_p + 122.5\,\epsilon_p^2 - 27.65\,\epsilon_p^3$$
$$C = -102.9\,\epsilon_p + 33.7\,\epsilon_p^2 + 137.8\,\epsilon_p^{0.1}$$

where

ϵ_c is the compressive strain during reverse loading, %
σ_c is the compressive stress during reverse loading, ksi
ϵ_p is the tensile prestrain, %

This equation is valid for values of compressive strain and tensile prestrain up to 1.5%

For $\epsilon_p < 0.1$, $\sigma_c = 300\,\epsilon_c$

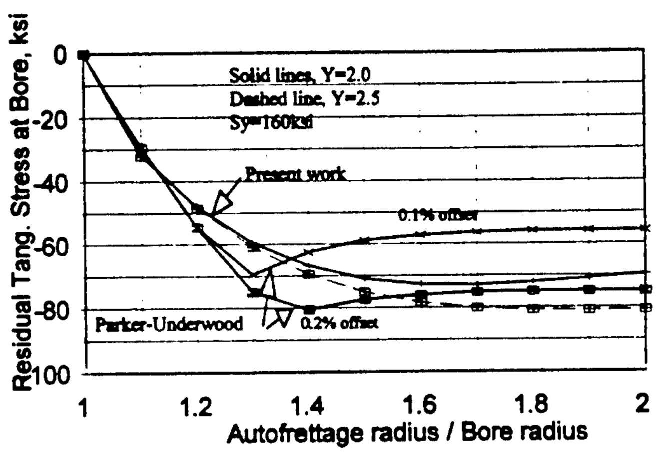
We can use equation (1) to estimate the tangential residual stress at the inner surface of a cylinder autofrettaged to a given amount of overstrain, including the effect of strain hardening during unloading. We calculate the amount of plastic tensile strain at the inner surface during autofrettage, which is equal to the tensile prestrain, ϵ_p, using the method given by Parker and Underwood. From this we can calculate the values of A, B, & C, which are used in equation (1) to obtain a compressive stress-strain curve.

To use this curve to estimate the amount of strain hardening that will occur during reverse yielding we assume that, during reverse yielding, this curve represents the relationship between the stress intensity and the tangential strain. This is not strictly correct but may not result in serious errors since, as the pressure approaches zero, the magnitudes of the radial and longitudinal stresses are small relative to that of the residual tangential stress.

The curve fit of Milligan's results indicates that the material remains elastic, during unloading, until the stress equals -30 ksi, regardless of the amount of tensile prestrain. We can then calculate the value of tangential strain at the point at which the stress intensity (difference between tangential and radial stress) at the inner surface equals -30 ksi using elastic equations. Actually, stress intensity does not have a sign, but we may use the negative sign to indicate that the tangential stress is compressive. We then assume that this point is equivalent to a point on the compressive stress-strain curve at which the stress equals -30 ksi and the strain equals -0.1%, assuming an elastic modulus of 30 million. We can also calculate the unloading strain at the inner surface assuming completely elastic unloading and assume that the final unloading strain with non-linear unloading will be the same as that for linear unloading. The change in strain from the point at which the stress intensity equals -30 ksi to the above total unloading strain is added to the assumed elastic unloading strain at the point at which the stress intensity equals 30 ksi, which is 0.1%. This strain value is used in equation (1) to calculate the final residual stress.

The results of this calculation are shown in the figure below., which compares these results with those from Parker and Underwood, for both the 0.1% and 0.2% offset yield values from Milligan's results. Also shown are the results of the proposed method for a diameter ratio, Y, of 2.5

The results of the above calculations give residual stress values which are similar to those obtained by Parker and Underwood



for the 0.2% offset compressive yield strengths. However, they do not show a significant decrease in residual stress magnitude with increasing overstrain, as shown by Parker and Underwood. They also show increasing residual stress magnitudes with increasing diameter ratio.

The important point of this discussion is to show that, although autofrettage has been used for many years, there is still no way to accurately calculate the residual stresses produced by autofrettage. It is hoped that this discussion will prompt someone to develop an elastic-plastic finite element method for accurately predicting the influence of the Bauschinger Effect on residual stresses produced by any manufacturing process involving plastic deformation of metals.

REFERENCE

[1] Milligan, R.V., et al, Trans. ASME, D, 480-488, June 1966 (see reference [5] of Parker and Underwood)

THE BAUSCHINGER EFFECT IN AUTOFRETTAGED TUBES - A COMPARISON OF MODELS INCLUDING THE ASME CODE. Anthony P. Parker and John H. Underwood

Authors' Closure to Discussion presented by D P Kendall

We appreciate the acknowledgement in the Kendall discussion that Division 3 of Section VIII of the ASME Boiler and Pressure Vessel Code may be non-conservative. We make no special claims for our model other than those stated in the paper, namely 'the authors' model [for bore hoop stresses], based upon 0.1% offset clearly provides .... a lower (conservative) bound' and '..... use of the ASME code distributions could potentially result in a very significant over-estimate of [fatigue] lifetimes'.

We emphasize yet again the differences between the data and models presented in Fig. 2, in particular the very significant deviation from available experimental data of the ASME code and the close proximity of the experimental data to our 0.1% offset model results.

The present ASME code model is not shown in the discussion but by reference to Fig. 2 the new 'present work' profile is clearly well removed from both the ASME code and the bulk of the experimental data, particularly at values of autofrettage radius / bore radius exceeding 1.5 or thereabouts.

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